Available online at www.sciencedirect.com
SCIENCE DIRECT®

Physics Letters B 567 (2003) 53–60

PHYSICS LETTERS B

www.elsevier.com/locate/npe

On the supermultiplet of anomalous currents in $d = 6$

R. Manvelyan [1], W. Rühl

Department of Physics, Theoretical Physics, University of Kaiserslautern, P.O. Box 3049, 67653 Kaiserslautern, Germany

Received 27 May 2003; accepted 22 June 2003

Editor: P.V. Landshoff

Abstract

The multiplet of superconformal anomalous currents in the case $(1,0)$, $d = 6$ is derived. The supersymmetric multiplet of anomalies contains the trace of the energy–momentum tensor, the gamma trace of the supercurrent and some topological vector current with divergence equal to the R-current anomaly. The extension of this consideration to the $(2,0)$ case and some application and motivation coming from AdS_7/CFT_6 correspondence is discussed.

1. Introduction

AdS/CFT correspondence [1] provides a new tool for understanding the strong coupling dynamics of gauge theories. The most developed example is IIB string/gravity on $AdS_5 \times S^5$ describing the large N dynamics of $\mathcal{N} = 4$ Super-Yang–Mills theory. The investigation of the anomalies [2] and the correlation functions in this approach [3,4] is very important because their universal behavior with respect to the renormalization from the weak to the strong coupling regime [5] offers a unique possibility to test this correspondence using the free field approach in the gauge theory calculations. Another interesting example of this correspondence is AdS_7/CFT_6 case describing duality between the M-theory/11dSUGRA and the maximal supersymmetric $(2,0)$ tensor multiplet in $d = 6$ [6]. This theory deals with the low energy regime of N coincident M5-branes and has some mysterious properties like the absence of a free coupling parameter, the difficulties in the lagrangian formulation of the self-dual antisymmetric tensor field and its nonabelian generalization. The test of AdS_7/CFT_6 includes the investigation of two- and three-point correlation functions of the energy–momentum tensor and vector R-currents [7,8], renormalization properties of trace anomaly coefficients in external gravitational and vector fields [9,10] and the behavior of the R-current anomaly from the weak to the strong coupling limit [11]. These investigations lead to some discrepancy in contrast to the AdS_5/CFT_4 case concerning the behavior of the coefficient of the Euler density term of the trace anomaly [9, 12] and the general structure of the R-anomaly in different energy limits [11]. These phenomena still need further explanations. In this Letter we investigate the SUSY structure of the superconformal current multiplet in $d = 6$ in the $(1,0)$ case and discuss the $(2,0)$ current and anomaly multiplets also. Our goal is to construct the multiplet

E-mail addresses: manvel@moon.yerphi.am, manvel@physik.uni-kl.de (R. Manvelyan), ruehl@physik.uni-kl.de (W. Rühl).

[1] On leave from Yerevan Physics Institute.

0370-2693
doi:10.1016/j.physletb.2003.06.014

of anomalies including the trace and the R-current anomalies as bosonic components in $d=6$ using *rigid* SUSY algebra. This can help to understand the strange behavior of the anomalous coefficients during renormalization from the weak (free fields) to strong (AdS/CFT) coupling. The result described below is the following. *The anomaly multiplet in $d=6$ includes in one superfield the trace anomaly of the energy–momentum tensor, the gamma-trace of the supersymmetry current and the topological Chern–Simons current whose divergence is equal to the R-current anomaly.*

2. Superconformal currents in the $(1,0)$ case and the anomaly multiplet

The conserved conformal supercurrent in the case $d=6$, $(1,0)$ case forms the scalar superfield J satisfying the third order conservation condition[2] [13]

$$D^{3\alpha ijk}J=\epsilon^{\alpha\beta\gamma\delta}D^{(i}_{\beta}D^{j}_{\gamma}D^{k)}_{\delta}J=0. \tag{1}$$

From this equation and the condition of closure of the SUSY algebra one can derive (see Ref. [13]) the independent components, their transformation and conservation rules. Here we will only list the independent components in ascending order of Weyl weights (dimensions) and present the conservation conditions.

J	off-shell scalar	auxiliary field
ψ^i_α	off-shell fermion	auxiliary field
$C_{\alpha\beta}$	$C_{\alpha\beta}=C_{(\alpha\beta)}$	off-shell auxiliary self-dual third rank tensor
$V^{ij}_{\alpha\beta}$	$V^{ij}_{\alpha\beta}=V^{(ij)}_{[\alpha\beta]}$, $\partial^{\alpha\beta}V^{ij}_{\alpha\beta}=0$	conserved $SU(2)$-triplet R-symmetry current
$S^i_{\gamma,\beta\alpha}$	$S^i_{\gamma,\beta\alpha}=S^i_{\gamma,[\beta\alpha]}$, $S^i_{[\gamma,\beta\alpha]}=\Sigma^{a\alpha\beta}S^i_{a,\beta}=0$, $\partial^{\beta\alpha}S^i_{\gamma,\beta\alpha}=0$	conformal, conserved supersymmetry current
$T_{\delta\gamma,\beta\alpha}$	$T_{\delta\gamma,\beta\alpha}=T_{[\delta\gamma],[\beta\alpha]}=T_{\beta\alpha,\delta\gamma}$, $\partial^{\beta\alpha}T_{\beta\alpha,\delta\gamma}=0$, $T_{\delta[\gamma,\beta\alpha]}=T_{\beta\alpha}{}^{\beta\alpha}=0$	conformal, conserved energy–momentum tensor

(2)

We want to deform (1) with some off-shell supermultiplet of anomalies

$$D^{3\alpha ijk}J=A^{\alpha ijk}, \tag{3}$$

but with structure maintaining conservation of the supersymmetry current and energy–momentum tensor

$$\partial^{\beta\alpha}S^i_{\gamma,\beta\alpha}=0,\qquad \partial^{\beta\alpha}T_{\beta\alpha,\delta\gamma}=0 \tag{4}$$

and violating the tracelessness and gamma tracelessness conditions of the latter and the conservation of the vector $SU(2)$ R-current

$$\epsilon^{\alpha\beta\gamma\delta}S^i_{\beta,\gamma\delta}\neq 0,\qquad T_{\alpha\beta}{}^{\alpha\beta}\neq 0,\qquad \partial^{\alpha\beta}V^{ij}_{\alpha\beta}\neq 0. \tag{5}$$

More precisely, we want to find a superfield with the last two levels (with Weyl weight $11/2$ and 6) of independent components

$$\ldots;\quad \xi^{i\alpha};\quad \Theta,\quad v^{ij}=v^{(ij)} \tag{6}$$

[2] Our conventions and notations can be found in the appendix.

which we can use for parametrization of the general anomaly superfield (3) and which leads to the violation of the conservation conditions (5) (the analogous consideration for the gauge and gravitational anomalies in $(1,0)$ $d=6$ case can be found in Ref. [15]). First of all we have to write a possible transformation of the set of fields (6)

$$D^j_\beta \xi^{i\alpha} = \delta^\alpha_\beta\big(\varepsilon^{ji}\Theta + v^{ji}\big) + M^\alpha_\beta \varepsilon^{ji} + M^{\alpha(ji)}_\beta + \cdots, \tag{7}$$

$$D^i_\alpha \Theta = -\frac{a}{2} i\partial_{\alpha\beta}\xi^{i\beta}, \tag{8}$$

$$D^k_\alpha v^{ji} = -b\varepsilon^{k(j} i\partial_{\alpha\beta}\xi^{i)\beta}, \tag{9}$$

$$M^\alpha_\alpha = M^{\alpha(ji)}_\alpha = 0, \tag{10}$$

here dots in (7) replace derivative terms of unknown auxiliary fields with lower Weyl weights. The tracelessness conditions (10) for the general antisymmetric and symmetric part of transformation of $\xi^{i\alpha}$ are natural because we already extracted the δ^α_β trace of $D^j_\beta \xi^{i\alpha}$ in (7) as a bracket with our anomalies. Note that the presence in r.h.s. of Eqs. (8) and (9) of only the derivatives of the lower spinor components expresses the lack of independent components of the anomaly multiplet with higher Weyl weights. Then we can try to fix the unknown coefficients a, b and possible configurations for the second rank antisymmetric tensor fields M^α_β and $M^{\alpha(ij)}_\beta$. Immediately we obtain the following restrictions

$$\big\{D^i_\alpha, D^j_\beta\big\}\Theta = i\varepsilon^{ij}\partial_{\alpha\beta}\Theta \Rightarrow \partial_{\lambda(\alpha} M^{\lambda(ij)}_{\beta)} = 0, \qquad \partial_{\lambda[\alpha} M^\lambda_{\beta]} = 0, \tag{11}$$

$$\big\{D^k_\alpha, D^l_\beta\big\}v^{ji} = i\varepsilon^{kl}\partial_{\alpha\beta} v^{ji} \Rightarrow \partial_{\lambda(\alpha} M^\lambda_{\beta)} = 0, \qquad \partial_{\lambda[\alpha} M^{\lambda(ij)}_{\beta]} = 0. \tag{12}$$

So we obtain for both tensor fields not only a Bianchi identity (symmetric second rank equations) leading to the corresponding vector field potentials, but get another two restrictions (antisymmetric second rank equations) leading to the on-shell conditions for these potentials. So we prove the statement: *there is no off-shell supermultiplet of fields containing the set of anomalies* (6) *as highest Weyl weight independent components.*

Nevertheless we will try to construct the analogy of the well-known relation in the case $N=1$, $d=4$ for the anomalous currents

$$\overline{D}^{\dot\alpha} J_{\alpha\dot\alpha} = D_\alpha A, \qquad \overline{D}_{\dot\alpha} A = 0 \tag{13}$$

looking for the possible multiplets with off-shell and absolutely unrestricted components (even without Bianchi identities or conservation condition). This type of multiplet could play the role of the chiral multiplet in $d=4$ (13) expressing anomalies in $N=1$, and also in $N=2$ cases. But in $d=6$ there is no chiral multiplet, instead we have here *only one known irreducible supermultiplet with unrestricted independent components*. It is the $SU(5)$ **5** superfield $L^{ijkl} = L^{(ijkl)}$ constrained with the following superfield condition

$$D^{(i}_\alpha L^{jklm)} = 0 \tag{14}$$

but containing only the off-shell and unconstrained components

$$\begin{aligned} &L^{ijkl}, \qquad \lambda^{jkl}_\alpha = D_{\alpha i} L^{ijkl}, \\ &G^{kl}_{\beta\alpha} = D_{\beta j}\lambda^{jkl}_\alpha, \qquad \xi^{i\delta} = \epsilon^{\gamma\beta\alpha\delta} D_{\gamma j} G^{ji}_{\beta\alpha}, \qquad \Theta = D_{\alpha i}\xi^{i\alpha}. \end{aligned} \tag{15}$$

This multiplet was considered in Refs. [13,17] and used in Ref. [15] for the construction of gauge anomalies in the $(1,0)$ case (the so-called relaxed hypermultiplet [16]). The same superfield we can use for the quantum deformation of the conservation relation (1) in the following way

$$D^{3\alpha ijk} J = -2\epsilon^{\alpha\gamma\beta\delta} i\partial_{\gamma\beta}\lambda^{ijk}_\delta = 4i\partial^{\beta\alpha} D_{\beta l} L^{lijk}. \tag{16}$$

So we express the general anomaly in (3) by mixing the conformal current multiplet (2) with the $SU(2)$ **5** superfield (15).

$$A^{\alpha ijk} = 4i\partial^{\beta\alpha} D_{\beta l} L^{lijk}. \tag{17}$$

Note that this equality itself concerns the usual derivative and does not lead directly to the relation between the traces of the current multiplet and components of L^{ijkl} (such as in $d=4$ case (13)). However exploring the relation (16) and the condition of closure of the supersymmetry we obtain the following relations for the transformations of the components and the set of conservation conditions:

$$J, \qquad L^{ijkl}; \tag{18}$$

$$D^i_\alpha J = \psi^i_\alpha, \qquad D^n_\alpha L^{ijkl} = \frac{4}{5}\varepsilon^{n(i}\lambda^{jkl)}_\alpha; \tag{19}$$

$$D^j_\beta \psi^i_\alpha = V^{ji}_{\beta\alpha} + \varepsilon^{ji} C_{\beta\alpha} + \frac{i}{2}\varepsilon^{ji}\partial_{\beta\alpha} J, \qquad D^i_\beta \lambda^{jkl}_\alpha = \frac{3}{4}\varepsilon^{i(j} G^{kl)}_{\beta\alpha} - \frac{5}{4}\partial_{\beta\alpha} L^{ijkl}; \tag{20}$$

$$\begin{aligned}
&D^k_\gamma V^{ji}_{\beta\alpha} = \varepsilon^{k(j} S^{i)}_{\gamma,\beta\alpha} - \frac{1}{9}\epsilon_{\gamma\beta\alpha\delta}\varepsilon^{k(j}\xi^{i)\delta} - 2i\partial_{[\gamma\beta}\lambda^{kji}_{\alpha]} + \frac{4}{5}i\varepsilon^{k(j}\partial_{\gamma[\beta}\psi^{i)}_{\alpha]} + \frac{1}{5}i\varepsilon^{k(j}\partial_{\beta\alpha}\psi^{i)}_\gamma,\\
&D^i_\gamma C_{\beta\alpha} = S^i_{(\alpha,\beta)\gamma} - \frac{4}{5}i\partial_{\gamma(\beta}\psi^i_{\alpha)}, \qquad \boxed{\partial^{\beta\alpha} S^i_{\gamma,\beta\alpha} = 0}, \qquad \boxed{S^i_{[\gamma,\beta\alpha]} = \tfrac{1}{9}\epsilon_{\gamma\beta\alpha\delta}\xi^{\delta i}},
\end{aligned} \tag{21}$$

$$\begin{aligned}
&D^k_\gamma G^{ji}_{\beta\alpha} = \frac{1}{9}\epsilon_{\gamma\beta\alpha\delta}\varepsilon^{k(j}\xi^{i)\delta} - \frac{1}{3}i\partial_{\beta\alpha}\lambda^{kji}_\gamma - \frac{8}{3}i\partial_{\gamma[\beta}\lambda^{kji}_{\alpha]};\\
&D^j_\delta S^i_{\gamma,\beta\alpha} = \frac{1}{2}\varepsilon^{ji} T_{\delta\gamma,\beta\alpha} + \frac{2}{3}i\partial_{\delta\gamma} V^{ji}_{\alpha\beta} + \frac{2}{15}i\partial_{\beta\alpha} V^{ji}_{\delta\gamma} + \frac{2}{15}i\partial_{\gamma[\beta} V^{ji}_{\alpha]\delta} + \frac{2}{3}i\partial_{\delta[\beta} V^{ji}_{\alpha]\gamma}\\
&\qquad + \frac{2}{3}i\partial_{\beta\alpha} G^{ji}_{\gamma\delta} + \frac{2}{3}i\partial_{\gamma\delta} G^{ji}_{\beta\alpha} + \frac{1}{3}i\partial_{\gamma[\beta} G^{ji}_{\alpha]\delta} - \frac{7}{3}i\partial_{\delta[\beta} G^{ji}_{\alpha]\gamma}\\
&\qquad + i\varepsilon^{ji}\partial_{\delta[\alpha} C_{\beta]\gamma} - \frac{1}{5}i\varepsilon^{ji}\partial_{\gamma[\alpha} C_{\beta]\delta} - \frac{1}{5}i\varepsilon^{ji}\partial_{\beta\alpha} C_{\delta\gamma}, \qquad \boxed{\partial^{\beta\alpha} V^{ji}_{\beta\alpha} + \partial^{\beta\alpha} G^{ji}_{\beta\alpha} = 0},
\end{aligned} \tag{22}$$

$$\begin{aligned}
&D^j_\beta \xi^{i\alpha} = \frac{1}{8}\varepsilon^{ji}\delta^\alpha_\beta \Theta + \frac{9}{2}i\partial^{\gamma\delta} G^{ji}_{\gamma\delta}\delta^\alpha_\beta - 6i\partial^{\gamma\alpha} G^{ji}_{\gamma\beta}, \qquad \boxed{\Theta + 3T_{\beta\alpha}{}^{\beta\alpha} = 0},\\
&D^i_\varepsilon T_{\delta\gamma,\beta\alpha} = -\frac{2}{3}i\partial_{[\delta[\alpha} S^i_{\varepsilon,\beta]\gamma]} - 2i\partial_{\varepsilon[\delta} S^i_{\gamma],\beta\alpha} - 2i\partial_{\varepsilon[\beta} S^i_{\alpha],\delta\gamma} - \frac{1}{3}i\partial_{\delta\gamma} S^i_{\varepsilon,\beta\alpha} - \frac{1}{3}i\partial_{\beta\alpha} S^i_{\varepsilon,\delta\gamma},\\
&D^i_\alpha \Theta = -4i\partial_{\alpha\beta}\xi^{i\beta}, \qquad \boxed{\partial^{\beta\alpha} T_{\beta\alpha,\gamma\delta} = 0},
\end{aligned} \tag{23}$$

where we mark the important conservation conditions and the relations between the previously independent components by using frames. Thus we obtain the following result: *The multiplet of anomalies in the six-dimensional case includes the trace of the energy–momentum tensor* (Θ)*, the gamma-trace of the supercurrent* ($\xi^{i\alpha}$) *and some* $SU(2)$ *triplet of the vector currents* $G^{ij}_{\alpha\beta}$ *restricted to express the R-symmetry anomaly by it's divergence*. But the R-symmetry anomaly is kind of the chiral anomalies and hence it is some linear combination of parity-odd topological invariants constructed from the external fields. Therefore, we can always express this type of anomaly in the form of the divergence of some topological vector currents dual to the Chern–Simons forms in one dimension less

$$^*(F\wedge F\wedge F) \sim \partial_a \operatorname{tr}\big(\epsilon^{abcdef} A_b F_{cd} F_{ef} + \cdots\big). \tag{24}$$

Changing to the usual notations we can say that the following objects are the components of the single superfield

$$\Theta = D^4_{ijkl} L^{ijkl} = 12 T^a_a, \tag{25}$$

$$\xi^{i\alpha} = D^{3\alpha}_{jkl} L^{ijk} = 3 S_{\beta,a}\Sigma^{a\beta\alpha}, \tag{26}$$

$$G^{ij}_{\alpha\beta} = D^2_{\alpha\beta,kl} L^{ijkl}, \quad \text{with } \partial^{\alpha\beta} G^{ij}_{\alpha\beta} = 4\partial^a V^{ij}_a. \tag{27}$$

Of course, these relations are dependent on the initial normalization of the currents in the superconformal current multiplet (2).

3. Discussion of superconformal currents and anomalies in $(2,0)$ case

First of all we note that the previous statement about the absence in the case of six-dimensional $(1,0)$ of an off-shell supermultiplet with highest independent weight components corresponding to the trace and R-symmetry anomalies is valid in the $(2,0)$ case also. We have just to replace all $SU(2)$ R-symmetry indices with $USp(4)$ and consider instead of M^{α}_{β} the antisymmetric tensor $M^{\alpha[ij]}_{\beta}$. We will come again to the on-shell condition from the closure of the SUSY algebra. Nevertheless in this case the supermultiplet of conserved conformal currents exists and is described again by the scalar superfield but now the **14** of the $USp(4)$ $J^{ij,kl}$, with the following set of properties and constraints

$$J^{ij,kl} = J^{[ij],[kl]} = J^{kl,ij}, \qquad J^{ij,kl}\Omega_{ij} = 0, \qquad J^{ij,kl}\epsilon_{ijkl} = 0, \tag{28}$$

$$D^m_\alpha J^{ij,kl} = \Omega^{m[i}\lambda^{j],kl}_\alpha + \frac{1}{4}\Omega^{ij}\lambda^{m,kl}_\alpha + \Omega^{m[k}\lambda^{l],ij}_\alpha + \frac{1}{4}\Omega^{kl}\lambda^{m,ij}_\alpha, \tag{29}$$

$$\lambda^{m,kl}_\alpha = \lambda^{m,[kl]}_\alpha, \qquad \lambda^{m,kl}_\alpha\Omega_{kl} = 0, \qquad \lambda^{m,kl}_\alpha\Omega_{mk} = 0. \tag{30}$$

The components of this conserved superconformal current multiplet of $(2,0)$ theory were listed for the first time in Ref. [13]

$J^{ij,kl}$	**14** of $USp(4)$off-shell scalars see (28)	auxiliary field	
$\lambda^{i,jk}_\alpha$	**16** of $USp(4)$ off-shell fermion see (30)	auxiliary field	
$C^{ij}_{\alpha\beta}$	$C^{ij}_{\alpha\beta} = C^{[ij]}_{(\alpha\beta)}$	off-shell auxiliary self-dual	
	$C^{ij}_{\alpha\beta}\Omega_{ij} = 0$	third rank tensor	
$V^{ij}_{\alpha\beta}$	$V^{ij}_{\alpha\beta} = V^{(ij)}_{[\alpha\beta]}$,	conserved **10** of $USp(4)$	(31)
	$\partial^{\alpha\beta}V^{ij}_{\alpha\beta} = 0$	R-symmetry current	
$S^i_{\gamma,\beta\alpha}$	$S^i_{\gamma,\beta\alpha} = S^i_{\gamma,[\beta\alpha]}$, $S^i_{[\gamma,\beta\alpha]} = \Sigma^{a\alpha\beta}S^i_{a,\beta} = 0$	conformal, conserved	
	$\partial^{\beta\alpha}S^i_{\gamma,\beta\alpha} = 0$	supersymmetry current	
$T_{\delta\gamma,\beta\alpha}$	$T_{\delta\gamma,\beta\alpha} = T_{[\delta\gamma],[\beta\alpha]} = T_{\beta\alpha,\delta\gamma}$,	conformal, conserved	
	$\partial^{\beta\alpha}T_{\beta\alpha,\delta\gamma} = 0$, $T_{\delta[\gamma,\beta\alpha]} = T_{\beta\alpha}{}^{\beta\alpha} = 0$	energy–momentum tensor	

and the transformation rules and the coupling with conformal supergravity were considered in Ref. [18]. Following the previous section we could try to deform the superfield conservation condition (29) introducing the superfield $A^{m,[ij],[kl]}_\alpha$ expressed through some *off-shell* multiplet of anomalies containing a topological Chern–Simons current $G^{(ij)}_{[\alpha\beta]}$, $\partial^{\alpha\beta}G^{ij}_{\alpha\beta} \sim \partial^{\alpha\beta}V^{(ij)}_{\alpha\beta}$ and the supersymmetry and the trace anomalies $\xi^{i\alpha}$ and Θ. Unfortunately the structure of the off-shell multiplets in the maximal extended $(2,0)$ case is not well understood and we cannot present here the full solution of this problem, but a partial answer is the following.

$$D^i_\alpha\Theta = -\frac{1}{2}i\partial_{\alpha\beta}\xi^{i\beta}, \tag{32}$$

$$D^j_\beta\xi^{i\alpha} = \delta^\alpha_\beta\Omega^{ji}\Theta - \frac{6}{5}i\partial_{\beta\gamma}G^{\alpha\gamma ji} - \frac{1}{5}\delta^\alpha_\beta i\partial_{\gamma\delta}G^{\gamma\delta ji} - \frac{4}{5}i\partial_{\beta\gamma}C^{\alpha\gamma ji}, \tag{33}$$

$$G^{\alpha\gamma ji} = G^{[\alpha\gamma](ji)}, \qquad C^{\alpha\gamma ji} = C^{(\alpha\gamma)[ji]}, \qquad C^{\alpha\gamma ji}\Omega_{ji} = 0,$$

$$D^k_\gamma G^{\beta\alpha ij} = \delta^{[\beta}_\gamma\xi^{\alpha](i}\Omega^{j)k} + \cdots, \tag{34}$$

$$D^k_\gamma C^{\beta\alpha ij} = \delta^{(\beta}_\gamma \xi^{\alpha)[j}\Omega^{i]k} + \frac{1}{4}\delta^{(\beta}_\alpha \xi^{\alpha)k}\Omega^{ji} + \cdots. \tag{35}$$

Note that the coefficients in (32) and (33) and the presence of the new auxiliary tensor field $C^{\alpha\beta[ij]}$ are necessary from the condition of the closure of supersymmetry algebra on Θ and $\xi^{i\alpha}$. Unfortunately we cannot present the complete tower of auxiliary fields,[3] but can predict that if this supermultiplet of anomalies exists, the relation between different anomalies has to remain in the same way as in (25)–(27). For comparison let us remember the $d = 4$ $N = 2$ and $N = 1$ cases. In the $N = 2$ case the conformal conserved currents and anomalies described by just a scalar real superfield and $N = 2$ chiral multiplet correspondingly (see, e.g., [19] and references there) with the following anomalous relation

$$D^{ij} J = \overline{D}^{ij} S, \qquad \overline{D}^i_{\dot\alpha} S = 0, \qquad D^{ij} = D^i_\alpha D^{j\alpha} \tag{36}$$

The $N = 2$ anomaly multiplet S is in the language of $N = 1$ superfields equivalent to the chiral scalar and vector multiplet. So the $N = 1$ anomaly multiplet A in Eq. (13) can be identified with this scalar chiral part of the $N = 2$ chiral superfield. It means that the trace anomaly and $U(1)$ part of $SU(2)$ R-symmetry anomaly of the $N = 2$ theory form this $N = 1$ superfield with the same relative coefficients, and extension of SUSY just adds a deformation to the conservation laws of additional components of supercurrent and R-current. Of course, as an artefact of this splitting to $N = 1$ superfields we will get the Konishi anomaly.

Something like this we can expect in six dimensions. It means that our $(1,0)$ anomaly multiplet L^{ijkl} could be some part of an unknown $(2,0)$ anomaly multiplet superfield if we will consider that in the language of the $(1,0)$ superfields. This consideration is in progress and will be studied elsewhere. The concrete application of our result to the explanation of the behavior of the anomaly coefficients of $(2,0)$ multiplet also needs additional considerations. Another interesting task connected with the latter is to express the anomaly superfield through the linearized external conformal supergravity (so-called Weyl multiplet) and find the superfield generalization of the Euler density and the Weyl invariant combination of curvature [2] which in $d = 6$ possibly contribute to the conformal anomaly.

Acknowledgements

Work of R.M. supported by DFG (Deutsche Forschungsgemeinschaft) and partially supported by the Volkswagen Foundation of Germany. The authors would like to thank R. Mkrtchyan, R. Minasian and T. Leonhardt for discussions.

Appendix A. Notation and convention

We use the normalized antisymmetrization $[\ldots]$ and symmetrization $(\ldots)$ of all type of indices. Latin indices $a, b, c, \ldots = 1, 2, \ldots, 6$ we use for usual $SO(1,5)$ tensors and Greek indices $\alpha, \beta, \gamma, \ldots = 1, 2, 3, 4$ for spinor $SU(4)$. The convention for $d = 6$ symplectic Majorana–Weyl spinors is [14]:

$$\left(\psi^i_\alpha\right)^* \equiv \Omega_{ij} B^\beta_{\dot\alpha} \psi^j_\beta, \qquad B^* B = -1. \tag{A.1}$$

Here in the $(1,0)$ case R-symmetry group indices $i, j, k, \ldots$ belong to $SU(2)$, $(i, j, \ldots = 1, 2)$ and the corresponding invariant symplectic metric $\Omega_{ij} = \varepsilon_{ij}$. In the $(2,0)$ case the corresponding R-symmetry group is $USp(4)$, $(i, j, \ldots = 1, 2, 3, 4)$ and we have two symplectic tensors Ω_{ij} and $\epsilon_{ijkl} = 3\Omega_{i[j}\Omega_{kl]}$. But in both cases the existence of the unitary matrix B (A.1) which transfers dotted and undotted indices allows us to formulate

[3] The dots in (34) and (35) mean the possible derivatives of the next fermion field $\psi^{i[jk],\gamma\beta\alpha}$.

everything without dotted $SU(4)^*$ indices using only $d=6$ chiral SUSY algebra

$$\{D^i_\alpha, D^j_\beta\} = i\Omega^{ij}\partial_{\alpha\beta} \quad (\Omega^{ij} = \varepsilon^{ij} \text{ in the } (1,0) \text{ case}). \tag{A.2}$$

$\partial_{\alpha\beta} = \Sigma^a_{\alpha\beta}\partial_a$ and $\Sigma^a_{\alpha\beta}$ is the set of 6 antisymmetric sigma matrices coming from the definition of usual $d=6$ gamma matrices in the following way with the corresponding normalization and completeness rules [13]

$$\Sigma^a_{\alpha\beta} = -B^{\dot\beta}_{\beta}\Sigma^a_{\alpha\dot\beta} = -\Sigma^a_{\beta\alpha}, \tag{A.3}$$

$$\Gamma^a = \begin{pmatrix} 0 & \Sigma^a_{\alpha\dot\beta} \\ \widetilde{\Sigma}^{a\dot\alpha\beta} & 0 \end{pmatrix}, \tag{A.4}$$

$$\Sigma^a_{\alpha\beta}\Sigma^{b\alpha\beta} = -4\eta^{ab}, \qquad \eta^{ab} = (+,-----), \tag{A.5}$$

$$\Sigma^a_{\alpha\beta}\Sigma_{a\gamma\delta} = -2\epsilon_{\alpha\beta\gamma\delta}. \tag{A.6}$$

Following this definition we can present a table of conversion and rules for corresponding tensors and spin-tensors.

V_a	$V_{\alpha\beta} = -V_{\beta\alpha}$
$F_{ab} = -F_{ba}$	$F^\alpha_\beta,\ F^\alpha_\alpha = 0$
$G_{abc} = G_{[abc]} = {}^*G_{abc}$	$G_{\alpha\beta} = G_{(\alpha\beta)}$
$G_{abc} = G_{[abc]} = -{}^*G_{abc}$	$G^{\alpha\beta} = G^{(\alpha\beta)}$
$T_{ab} = T_{(ba)},\ T^a_a = 0$	$T_{\alpha\beta,\gamma\delta} = T_{\gamma\delta,\alpha\beta} = T_{[\alpha\beta],[\gamma\delta]},\ T_{[\alpha\beta,\gamma\delta]} = T_{\alpha[\beta,\gamma\delta]} = 0$
$(S_a)_\alpha,\ (\Sigma^a S_a)^\alpha = 0$	$S_{\alpha,\beta\gamma} = S_{\alpha,[\beta\gamma]},\ S_{[\alpha,\beta\gamma]} = 0$

In addition we will present some useful formulas and relations

$$V^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}V_{\gamma\delta}, \qquad V_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\gamma\delta}V^{\gamma\delta}, \tag{A.7}$$

$$\partial_{[\varepsilon\gamma}V_{\beta\alpha]} = \frac{1}{12}\epsilon_{\varepsilon\gamma\beta\alpha}\partial^{\mu\nu}V_{\mu\nu}, \qquad \partial_{[\varepsilon\lambda}\delta^\delta_{\mu]} = \frac{1}{3}\epsilon_{\mu\varepsilon\lambda\gamma}\partial^{\delta\gamma}, \tag{A.8}$$

$$\partial_{\mu\nu}\partial^{\lambda\nu} = \frac{1}{4}\partial_{\mu\nu}\partial^{\mu\nu}\delta^\lambda_\mu, \qquad \Box = -\partial^a\partial_a = \frac{1}{4}\partial_{\mu\nu}\partial^{\mu\nu}. \tag{A.9}$$

For the $(1,0)$ case we used $SU(2)$ index rules which are presented here

$$V^i = \varepsilon^{ij}V_j, \qquad V_i = V^j\varepsilon_{ji}, \tag{A.10}$$

$$\varepsilon^{ij}\varepsilon_{ik} = \delta^i_k, \qquad \varepsilon^{[ij}V^{k]} = 0, \tag{A.11}$$

$$V^{k(i}\varepsilon^{j)l} - V^{l(i}\varepsilon^{j)k} = V^{ij}\varepsilon^{kl}, \qquad V^{ij} = V^{(ij)}, \tag{A.12}$$

$$V^{(ij}\varepsilon^{k)l} - V^{(ij}\varepsilon^{l)k} = \frac{4}{3}V^{ij}\varepsilon^{kl}. \tag{A.13}$$

Some formulas for the $(2,0)$ case ($USp(4)$ R-symmetry case) are:

$$V^i = \Omega^{ij}V_j, \qquad V_i = V^j\Omega_{ji}, \tag{A.14}$$

$$\Omega_{i[j}\Omega_{kl]} = \frac{1}{3}\epsilon^{ijkl}, \qquad \epsilon^{[ijkl}V^{m]} = 0, \tag{A.15}$$

$$\Omega^{[ij}\Phi^{kl]} = \frac{1}{2}\left(\Omega^{[ij}\Phi^{k]l} - \Omega^{l[k}\Phi^{ij]}\right) = 0, \tag{A.16}$$

$$\text{if } \Phi^{ij} = \Phi^{[ij]} \quad \text{and} \quad \Phi^{ij}\Omega_{ij} = 0. \tag{A.17}$$

Multiplications of the derivative operators give:

$$D^i_\alpha D^j_\beta = \frac{i}{2}\varepsilon^{ij}\partial_{\alpha\beta} + \varepsilon^{ij} D_{(\alpha\beta)} + D^{(ij)}_{[\alpha\beta]} \quad \text{for } (1,0), \tag{A.18}$$

$$D^i_\alpha D^j_\beta = \frac{i}{2}\Omega^{ij}\partial_{\alpha\beta} + \Omega^{ij} D_{(\alpha\beta)} + D^{(ij)}_{[\alpha\beta]} + D^{[ij]}_{(\alpha\beta)}, \tag{A.19}$$

$$\Omega_{ij} D^{[ij]}_{(\alpha\beta)} = 0 \quad \text{for } (2,0).$$

References

[1] J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231, hep-th/9711200;
S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Phys. Lett. B 428 (1998) 105, hep-th/9802109;
E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253, hep-th/9802150;
O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri, Y. Oz, Phys. Rep. 323 (2000) 183, hep-th/9905111.
[2] M.J. Duff, Nucl. Phys. B 125 (1977) 334;
M.J. Duff, Class. Quantum Grav. 11 (1994) 1387, hep-th/9308075;
S. Deser, M.J. Duff, C.J. Isham, Nucl. Phys. B 111 (1976) 45;
L. Bonora, P. Pasti, M. Bregola, Class. Quantum Grav. 3 (1986) 635;
S. Deser, A. Schwimmer, Phys. Lett. B 309 (1993) 279, hep-th/9302047;
D.R. Karakhanian, R.P. Manvelian, R.L. Mkrtchian, Mod. Phys. Lett. A 11 (1996) 409, hep-th/9411068;
T. Arakelian, D.R. Karakhanian, R.P. Manvelian, R.L. Mkrtchian, Phys. Lett. B 353 (1995) 52.
[3] D.Z. Freedman, S.D. Mathur, A. Matusis, L. Rastelli, Nucl. Phys. B 546 (1999) 96, hep-th/9804058;
G. Chalmers, H. Nastase, K. Schalm, R. Siebelink, Nucl. Phys. B 540 (1999) 247, hep-th/9805105;
G. Arutyunov, S. Frolov, Phys. Rev. D 60 (1999) 026004, hep-th/9901121;
A. Bilal, C.-S. Chu, Nucl. Phys. B 562 (1999) 181, hep-th/9907106.
[4] M. Henningson, K. Skenderis, JHEP 9807 (1998) 023, hep-th/9806087;
M. Henningson, K. Skenderis, Fortschr. Phys. 48 (2000) 125, hep-th/9812032.
[5] A.C. Petkou, K. Skenderis, Nucl. Phys. B 562 (1999) 100, hep-th/9906030.
[6] P. Claus, R. Kallosh, A. Van Proeyen, Nucl. Phys. B 518 (1998) 117, hep-th/9711161;
E. Bergshoeff, E. Sezgin, A. Van Proeyen, Class. Quantum Grav. 16 (1999) 3193, hep-th/9904085.
[7] F. Bastianelli, S. Frolov, A.A. Tseytlin, Nucl. Phys. B 578 (2000) 139, hep-th/9911135.
[8] R. Manvelyan, A.C. Petkou, Phys. Lett. B 483 (2000) 264, hep-th/0003017.
[9] F. Bastianelli, S. Frolov, A.A. Tseytlin, JHEP 0002 (2000) 013, hep-th/0001041.
[10] R. Manvelyan, A.C. Petkou, JHEP 0006 (2000) 003.
[11] D. Freed, J.A. Harvey, R. Minasian, G. Moore, Adv. Theor. Math. Phys. 2 (1998) 601, hep-th/9803205;
J.A. Harvey, R. Minasian, G. Moore, JHEP 09 (1998) 4, hep-th/9808060;
K. Intriligator, Nucl. Phys. B 581 (2000) 257, hep-th/0001205.
[12] A.A. Tseytlin, Nucl. Phys. B 584 (2000) 233, hep-th/0005072.
[13] P.S. Howe, G. Sierra, P.K. Townsend, Nucl. Phys. B 221 (1983) 331.
[14] T. Kugo, P.K. Townsend, Nucl. Phys. B 221 (1983) 357.
[15] P.S. Howe, A. Umerski, Nucl. Phys. B 221 (1983) 331.
[16] P.S. Howe, K.S. Stelle, P.K. Townsend, Nucl. Phys. B 214 (1983) 519.
[17] J. Koller, Nucl. Phys. B 222 (1983) 319.
[18] E. Bergshoeff, E. Sezgin, A. Van Proeyen, Class. Quantum Grav. 16 (1999) 3193, hep-th/9904085.
[19] P.S. Howe, P.C. West, Nucl. Phys. B 486 (1997) 425, hep-th/9607239.